# Property–microstructure correlation in *in situ* formed Al<sub>2</sub>O<sub>3</sub>, TiB<sub>2</sub> and Al<sub>3</sub>Ti mixture-reinforced aluminium composites

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The *in situ* formed Al<sub>2</sub>O<sub>3</sub>, TiB<sub>2</sub> and Al<sub>3</sub>Ti mixture-reinforced aluminium composites were successfully fabricated by the reaction sintering of the TiO<sub>2</sub>–B–Al system in a vacuum. With increasing boron content in the TiO<sub>2</sub>–B–Al system, the amount of generated TiB<sub>2</sub> in the composites increased and Al<sub>3</sub>Ti content decreased. At the same time the distribution uniformity of the *in situ* formed Al<sub>2</sub>O<sub>3</sub> and TiB<sub>2</sub> particulates was obviously improved, and the size of the Al<sub>3</sub>Ti particles was reduced. The *in situ* Al<sub>2</sub>O<sub>3</sub> and TiB<sub>2</sub> particulates had sizes from 0.096–1.88 µm. The interface between the *in situ* formed particulates and the aluminium matrix was clean, and no consistent crystallographic orientation relationship was found. The strength and elastic modulus of the composites was changed from large fractured Al<sub>3</sub>Ti blocks and fine dimples, to fine dimples and pulled-out particulates. The strengthening and fracture of the composites have been modelled.

# 1. Introduction

Aluminium alloys discontinuously reinforced with ceramic particulates, whiskers or short fibres are currently being developed for various high-performance applications. To improve the interfacial compatibility and avoid serious interfacial reaction, various new processing techniques are being used to fabricate the high-performance composites. A process termed the XD<sup>™</sup> technique [1] has been developed to fabricate in situ ultrafine ceramic particle-reinforced metal matrix composites. The basic principle of this technique is that the ultrafine ceramic particles are formed in situ by the exothermal reaction between elements or between element and compound. Using this approach, the metal matrix composites composed of a wide variety of matrix materials (including aluminium, iron, copper, lead, nickel and titanium) and second-phase particles (including borides, carbides, nitrides and their mixtures) have now been produced [1]. The in situ composites exhibit improved strength at elevated temperature as well as enhanced wear and fatigue resistance [1, 2].

Recently,  $Al_2O_3$  particle-reinforced aluminium composite formed *in situ* through the reaction between TiO<sub>2</sub> and aluminium has been investigated [3, 4]. According to the reaction formula between TiO<sub>2</sub> and aluminium, if 1 volume of  $Al_2O_3$  is formed, about 2.26 volumes of  $Al_3$ Ti will be generated. The properties of the *in situ* formed  $Al_2O_3$  particle-reinforced aluminium composite are difficult to tailor or control due to the inevitable existence of a large amount of brittle Al<sub>3</sub>Ti intermetallic compound. In a previous paper [5], the *in situ* formation of  $Al_2O_3 \cdot Al_3Ti/Al$  and  $Al_2O_3 \cdot TiB_2/Al$  composites were investigated. It was indicated that the properties of the composites can be improved by eliminating  $Al_3Ti$  through the incorporation of boron. In present work, different contents of boron were incorporated into the TiO<sub>2</sub>-Al system, and the relationship between the properties and microstructure of the composites was studied.

# 2. Experimental procedure

Atomized aluminium powder (99.6% purity), TiO<sub>2</sub> powder (98.0% purity) and boron powder (99.0% purity) with an average size of 40, 3, 2 µm, respectively, were used as raw materials. The composites were fabricated by a powder metallurgy technique, whereby aluminium, TiO<sub>2</sub> and boron powders were ball-milled for 8 h. During powder blending, TiO<sub>2</sub> powder of the same weight was added in samples 8, 4, 5 and 3 in order to form 10.5 vol % Al<sub>2</sub>O<sub>3</sub> in the above composites, and different contents of boron powder were added to these four samples to give a B/TiO<sub>2</sub> molecular ratio of 0, 4/3, 5/3 and 2/1 in samples 8, 4, 5 and 3, respectively, so that the composites containing different contents of TiB<sub>2</sub> and Al<sub>3</sub>Ti could be produced. The cold-compacted powder mixture was heated to above 800 °C in a vacuum and maintain for 10 min, then cooled down to 600°C and hot-pressed. The pressed billets were extruded at an extrusion ratio of 20:1 at 420 °C. The metallographic observations and

X-ray diffraction analyses on the mechanically polished specimens were carried out. Sample 3 was solved in dilute hydrochloric acid; the solution with ceramic particulates was diluted and filtrated. The filtrated particulates were dried, then examined on a quantitative metallograph. The thin foils for transmission electron microscope (TEM) were prepared by the ion-milling technique. The foils were examined on a JEM 2000EX II high-resolution TEM (HRTEM). Tensile specimens, with a gauge diameter of 4 mm and a gauge length of 10 mm, were machined from the extruded rods, and tested at a strain rate of  $8.3 \times 10^{-4}$  s<sup>-1</sup>. The tensile fracture surfaces were observed on a scanning electron microscope (SEM). The elastic modulus of the composites was measured by a resonance method.

### 3. Results and discussion

In the  $TiO_2$ -B-Al system, the chemical reactions take place as follows

$$3\text{TiO}_2 + 4\text{Al} \rightarrow 2\text{Al}_2\text{O}_3 + 3[\text{Ti}] \tag{1}$$

 $[Ti] + 3Al \rightarrow Al_3Ti$  (2)

 $[Ti] + 2B \rightarrow TiB_2 \tag{3}$ 

All the reactions are exothermic. Therefore, when the temperature of the system reaches the starting temperature of the reaction between TiO<sub>2</sub> and aluminium, the heat released by the reaction rapidly raises the temperature of the system, and promotes the reactions between titanium and boron or aluminium. All reactions will be finished in a short time. According to the previous investigations on the Ti-B-Al system [2, 6], the reaction between titanium and boron occurs much more easily than that between titanium and aluminium. Therefore, there would be different contents of  $TiB_2$  and  $Al_3Ti$  in the composites with increasing boron content in the TiO<sub>2</sub>-B-Al system. If the boron in TiO<sub>2</sub>-B-Al system reacts completely with the titanium displaced from the reaction between TiO<sub>2</sub> and aluminium, different contents of TiB2 and Al3Ti in the composites can be generated (Table I). With increasing boron content in the TiO<sub>2</sub>-B-Al system, the amount of the TiB<sub>2</sub> generated increases and the Al<sub>3</sub>Ti content falls. When the molecular ratio of  $B/TiO_2$  is 2/1, the 9.5 vol %TiB<sub>2</sub> will be formed in situ and the generation of Al<sub>3</sub>Ti will be completely inhibited, provided that the titanium displaced from the reaction between TiO<sub>2</sub> and aluminium reacts completely with boron.

The X-ray diffractograph (XRD) of the composites is shown in Fig. 1. Aluminium,  $Al_2O_3$  and  $Al_3Ti$  peaks are seen in the XRD of sample 8 (Fig. 1a); indicating

that the reaction between TiO2 and aluminium has taken place to form Al<sub>3</sub>Ti. According to the reaction formula between TiO<sub>2</sub> and aluminium, the formation of 1 volume of  $Al_2O_3$  will be followed by 2.26 volumes of Al<sub>3</sub>Ti, so the intensity of the Al<sub>3</sub>Ti peaks in sample 8 is much higher than that of  $Al_2O_3$  peaks. When boron is incorporated into the TiO<sub>2</sub>-Al system, TiB<sub>2</sub> peaks appear and the intensity of the Al<sub>3</sub>Ti peaks is significantly weakened (Fig. 1b), which is further reduced with increasing content of boron in the  $TiO_2$ –B–Al system (Fig. 1c). When the molecular ratio of B/TiO<sub>2</sub> reaches 2/1, the Al<sub>3</sub>Ti diffraction lines disappear from the XRD (Fig. 1d), indicating that the Al<sub>3</sub>Ti is basically eliminated in sample 3. In the investigation on the Ti-B-Al system, the formation of TiB<sub>2</sub> was accompanied by a certain amount of Al<sub>3</sub>Ti [6]. These results indicate that the titanium displaced from fine  $TiO_2$  is much more active in the reaction between titanium and boron in comparison with the coarse titanium particles.

Metallographic observations indicated that the microstructure of the composites was quite different. In sample 8, the irregular white blocks with a size of about 20  $\mu$ m were Al<sub>3</sub>Ti, and their size was much greater than that of the added TiO<sub>2</sub> particles (Fig. 2a). The fine grey particulates with a size of about 1  $\mu$ m were Al<sub>2</sub>O<sub>3</sub>. The greyish zone was the aluminium matrix. It can be noted that the distribution of the Al<sub>3</sub>Ti blocks and Al<sub>2</sub>O<sub>3</sub> particulates in the aluminium matrix was obviously non-uniform; there were also



Figure 1 X-ray diffractographs of the composites: (a) sample 8, (b) sample 4, (c) sample 5, (d) sample 3. ( $\bullet$ ) Al<sub>2</sub>O<sub>3</sub>, (V) Al<sub>3</sub>Ti, ( $\blacktriangle$ ) TiB<sub>2</sub>, ( $\blacksquare$ ) Al.

TABLE I Conte	t of generated	phases (	vol %	6)
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Sample	Al <sub>3</sub> Ti	Al <sub>2</sub> O <sub>3</sub>	TiB <sub>2</sub>	$Al_2O_3 + TiB_2$	$Al_3Ti + Al_2O_3 + TiB_2$
8	23.73	10.50	0	10.50	34.23
4	7.92	10.50	6.34	16.84	24.76
5	3.95	10.50	7.91	18.41	22.36
3	0	10.50	9.50	20.00	20.00



Figure 2 Metallographs of the composites: (a) sample 8, (b) sample 4, (c) sample 5, (d) sample 3.

a few Al<sub>2</sub>O<sub>3</sub> particulates in Al<sub>3</sub>Ti blocks, and some micropores were also found. After boron was incorporated into the TiO<sub>2</sub>-Al system, the amount of Al<sub>3</sub>Ti blocks decreased, their size was reduced to about 10 µm, the amount of the fine particulates increased, and the distribution of the Al<sub>3</sub>Ti blocks and the fine particulates was improved (Fig. 2b). With increasing boron content to a  $B/TiO_2$  molecular ratio of 5/3 in the TiO<sub>2</sub>-B-Al system, the amount of Al<sub>3</sub>Ti blocks was further reduced (Fig. 2c). Al<sub>3</sub>Ti blocks with a size of about  $5 \,\mu m$  and fine particulates were uniformly distributed in the aluminium matrix. When the boron reached a B/TiO<sub>2</sub> molecular ratio of 2/1, the Al<sub>3</sub>Ti blocks disappeared and the distribution uniformity of the fine  $Al_2O_3$  and  $TiB_2$  particulates in the aluminium matrix was further improved (Fig. 2d).

Quantitative metallography examinations on the ceramic particulates taken from sample 3 indicated that the  $Al_2O_3$  and  $TiB_2$  particulates had an average size of  $0.31 \pm 0.04 \,\mu\text{m}$ ; the largest size of the particulates was  $1.88 \,\mu\text{m}$ , and the smallest size  $0.096 \,\mu\text{m}$ . Some much smaller particulates cannot be resolved by quantitative metallography.

TEM observations indicated that a large amount of the *in situ* formed particulates had a size of about 50 nm (Fig. 3), though quantitative metallographic examinations indicated that the particulates had a size



Figure 3 TEM image of sample 3.

from 0.096–1.88  $\mu$ m. The TiB<sub>2</sub> particulate in Fig. 4 is only 15 nm. These observations indicate that the *in* situ formed particulates are very fine, and their size is much less than that of *in situ* TiC particulates [7, 8] and TiB<sub>2</sub> particulates [2]. The ultrafine particulates could contribute to the increase in composite strength. The interface between the particulates and the aluminium matrix was clean and without a transitional layer between them. No consistent crystallographic



Figure 4 HRTEM image of sample 3.

orientation relationship between particulates and aluminium matrix was found.

Table II shows the properties of the composites. The tensile strength of sample 8 is only 145 MPa, and increases a little over that of the aluminium matrix (about 100 MPa), which demonstrates that the large amount of Al<sub>3</sub>Ti blocks existing in sample 8 have no obvious strengthening effect on the aluminium matrix. The ductility of the composite is also lower. When  $6.34 \text{ vol }\%\text{TiB}_2$  is formed and the content of Al<sub>3</sub>Ti is reduced to 7.91 vol% due to the addition of boron, the tensile strength of the composite is significantly improved and reaches 311 MPa. With increasing boron content in the  $TiO_2$ -B-Al system, the strength of the composite increases further. When the molecular ratio of the  $B/TiO_2$  is 2/1, the tensile strength of the composite (sample 3) reaches 381 MPa and increases by about 163% over that of sample 8 due to the elimination of the brittle Al<sub>3</sub>Ti blocks. With increasing the  $TiB_2$  content, the ductility of the composites can be improved by the reduction of the blocky Al<sub>3</sub>Ti in the composites, on the other hand, the ductility decreases due to the increase in the strength of the composites. Therefore, the ductility of the composites varies slightly, decreases first and then increases. The ductility of sample 3 with high strength is somewhat superior to that of sample 8, due to complete elimination of the brittle Al<sub>3</sub>Ti. The elastic modulus of sample 8 is only 78.9 GPa, a little above that of the aluminium matrix, indicating that a large amount of the blocky  $Al_3Ti$  in sample 8 does not increase the elastic modulus of the material. The slight increase in elastic modulus is attributed to 10.5 vol %Al<sub>2</sub>O<sub>3</sub> particulates. When the  $6.34 \text{ vol }\%\text{TiB}_2$  is formed and the content of the Al<sub>3</sub>Ti is reduced to 7.91 vol%, the elastic modulus of the composite (sample 4) reaches 107.0 GPa, and is significantly improved over that of sample 8.

Fig. 5 shows the temperature dependence of tensile strength for the composites. The strength of sample 8 decreases slowly and linearly with increasing test temperature. After incorporating boron into the TiO<sub>2</sub>-Al system, the tensile strength of the composites (samples 4, 5 and 3) is significantly improved at the temperatures ranging from room temperature to 400 °C, but decreases more rapidly than that of sample

TABLE II Properties of the composites

Sample	UTS (MPa)	YS (MPa)	El (%)	E (GPa)
8	145	110	5.42	78.9
4	311	271	4.83	107.0
5	328	301	4.70	_
3	381	340	5.53	-



Figure 5 Temperature dependence of tensile strength for the composites.

8 with increasing temperature, which indicates that the Al<sub>3</sub>Ti is somewhat beneficial to the elevated temperature strength of the composites, but the effect is small due to the large size of Al<sub>3</sub>Ti. The composites (samples 4, 5 and 3) exhibit a tensile strength at 300 °C, which is higher than that of sample 8 at room temperature.

SEM observations indicated that the tensile fracture surfaces of various composites were quite different. Many brittle fractured blocks with a size of about 20 µm were found on the fracture surface of sample 8, and secondary cracks were also found on the fractured blocks (Fig. 6a). The energy-dispersive X-ray analysis (EDAX) proved them to be Al<sub>3</sub>Ti. It is obvious that the brittle Al<sub>3</sub>Ti blocks are the weakest zones in the composites, their earlier brittle fracture results in the fracture of the composites. A few fine  $Al_2O_3$  particulates can be seen on the fractured Al<sub>3</sub>Ti blocks and in the fine dimples. After the content of Al<sub>3</sub>Ti was reduced to 7.91 vol %, the large fractured Al<sub>3</sub>Ti blocks were mostly eliminated (Fig. 6b). The tensile fracture surface of the composite (sample 4) mainly consisted of fine dimples. Only a small amount of the smaller fractured Al<sub>3</sub>Ti blocks can be found, indicating that the harmful effect of the blocky Al<sub>3</sub>Ti on the properties of the composite was mainly eliminated. The morphologies of the fracture surface of the composite varied little with decreasing Al<sub>3</sub>Ti content (Fig. 6c). When the molecular ratio of the  $B/TiO_2$  reached 2/1, no fractured Al<sub>3</sub>Ti blocks were found on the tensile fracture surface of the composite (sample 3), the fracture surface consisted of the fine dimples and a few



Figure 6 SEM images of tensile fracture surfaces: (a) sample 8, (b) sample 4, (c) sample 5, (d) sample 3.

particulates were pulled out (Fig. 6d). The fine dimple morphologies suggested a good interfacial bonding between particulate and matrix and indicated that the composite would have a high strength.

According to the literature [9], the yield strength in particulate composites is basically related to the particulate-dislocation interaction by means of the Orowan bowing mechanism [10]

$$\tau = \tau_s + \frac{T}{bL/2} \tag{1}$$

where  $\tau$  is shear yield stress,  $\tau_s$  the threshold shear stress associated with Orowan bowing, **b** the Burgers vector, T the line tension of a dislocation, and L the mean interparticle distance. L can be calculated from

$$L = \left(\frac{6}{\pi}V\right)^{-1/3} d_{\rm m} \tag{2}$$

where V is the volume fraction of the particulates and  $d_m$  is the mean particulate diameter. Then, the shear yield strength is given by

$$\tau = \tau_{\rm s} + \frac{2T}{bd_{\rm m}} \left(\frac{6}{\pi}V\right)^{1/3} \tag{3}$$

It is assumed that the increase in the yield strength of the composites is mainly due to the increase in  $TiB_2$ content and the Orowan mechanism plays a key role in the process. The shear yield strength of the composites in Equation 3 can be rewritten as

$$\tau = \tau_s + K V_t^{1/3} \tag{4}$$

where K stands for  $(2T/bd_m)(6/\pi)^{1/3}$ .  $V_t$  is the TiB<sub>2</sub> volume fraction. Because the diameter of the TiB<sub>2</sub> particulates does not change much, as observed, in a first approximation, the coefficient K could be considered as a constant.

According to Equation 4 the yield strength of the composites,  $\sigma_v$ , can be written as

$$\sigma_{\rm y} = \sigma_{\rm s} + C V_{\rm t}^{1/3} \tag{5}$$

where  $\sigma_s$  is the yield strength of the composite without TiB<sub>2</sub> and C is a constant. The best fit coefficients in Equation 5 could be found as

$$\sigma_{\rm v} = 110 + 97.15 \, V_{\rm t}^{1/3} \tag{6}$$

Fig. 7 shows the effect of  $TiB_2$  content on the yield strength of the composite. It is found that the calculated values in Equation 6 are in fair agreement with experimental results. For the present composites, the ultimate tensile strength is mainly affected by the fracture of large and brittle Al<sub>3</sub>Ti particles.

Quantitative metallographic studies on broken specimens as well as *in situ* observations indicated that the fracture stress of brittle particles embedded in a matrix follows the postulates of the Weibull statistics [11, 12]. The probability of fracture, F, for Al<sub>3</sub>Ti particles of volume V subjected to an average tensile stress  $\bar{\sigma}_p$  is given by [13]

$$F = 1 - \exp\left[\frac{-V}{V_0} \left(\frac{\bar{\sigma}_p}{\sigma_0}\right)^m\right]$$
(7)

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Figure 7 Effect of  $TiB_2$  content on the yield strength of the composites.

where *m* is the Weibull modulus,  $V_0$  and  $\sigma_0$  are two constants with dimensions of volume and stress, respectively. It is assumed that all the composites fracture under a certain probability of fracture of Al<sub>3</sub>Ti particles, then

$$\exp\left[\frac{-V}{V_0} \left(\frac{\bar{\sigma}_p}{\sigma_0}\right)^m\right] = c \tag{8a}$$

or

$$\bar{\sigma}_{\rm p} = C V^{-1/m} \tag{8b}$$

where c and C are constants.

Because the fracture of  $Al_3$ Ti particles leads to the fracture of the composites, the ultimate tensile strength of the composites,  $\sigma_e$ , is proportional to the average tensile stress,  $\bar{\sigma}_p$ , acting on the Al<sub>3</sub>Ti particles, hence

$$\sigma_{\rm c} = K V^{-1/m} \tag{9}$$

The *m* values are usually between 1 and 6 [12]; here, an average m = 3 is used. The best fit coefficient in Equation 9 could be found as

$$\sigma_{\rm c} = 518.4 \, V^{-1/3} \tag{10}$$

Fig. 8 shows the effect of the Al<sub>3</sub>Ti content on ultimate tensile strength of the composites. The calculated values in Equation 10 are basically in fair agreement with the experimental ones. However, when the content of Al<sub>3</sub>Ti particles is very low  $(V \rightarrow 0)$ , the equation is not applicable, because the high stress results in the fracture of the composite matrix preferentially, as shown in Fig. 8 by the dotted line.

Although the present models do not give a precise prediction of the relationship between the strength and volume fraction of particles, the general trends, i.e. the yield strength increase with increasing  $TiB_2$  contents, the ultimate tensile strength decrease with  $Al_3Ti$  content increase, are well modelled by Orowan dislocation bowing and fracture of brittle particles mechanisms, respectively.



Figure 8 Effect of  $Al_3Ti$  content on the tensile strength of the composites.

Because the yield and fracture behaviours are very complicated in this kind composite, further work is still required.

#### 4. Conclusions

1. With increasing boron content in the  $TiO_2-B-A1$  system, the amount of *in situ* formed  $TiB_2$  particulates increases and the large  $Al_3Ti$  blocks are refined and disappear gradually; the distribution uniformity of the *in situ* formed  $Al_2O_3$  and  $TiB_2$  particulates is obviously improved.

2. When the molecular ratio of the  $B/TiO_2$  in the  $TiO_2$ -B-Al system reaches 2/1, the Al<sub>3</sub>Ti in the composite can be completely eliminated, and the Al<sub>2</sub>O<sub>3</sub> and TiB<sub>2</sub> particulates with an average size of 0.31 µm are uniformly distributed in the aluminium matrix.

3. The interface between the *in situ* formed particulate and aluminium matrix is clean. No consistent crystallographic orientation relationship is found.

4. A large amount of  $Al_3Ti$  in the composite has no obvious beneficial effect on the strength and modulus of the composite. The strength and modulus can be significantly improved by incorporating boron into the TiO<sub>2</sub>-Al system, and the ductility varies slightly.

5. The composites exhibit the excellent elevated temperature strength up to  $300 \,^{\circ}\text{C}$  after incorporating boron into the TiO<sub>2</sub>-Al system.

6. With increasing  $TiB_2$  content in the composites, the morphologies of the tensile fracture surfaces of the composites change from large fractured  $Al_3Ti$  blocks and fine dimples to fine dimples and pulled out particulates.

7. With increasing  $TiB_2$  content in the composites, the yield strength of the composite increases, and the tensile strength decreases as  $Al_3Ti$  content increases, which is modelled by Orowan dislocation bowing and fracture of brittle particles mechanisms, respectively.

#### Acknowledgement

This work was supported by a grant from the Director of Foundation Institute of Metal Research, Chinese Academy of Sciences, to whom we are very grateful.

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Received 27 June 1994 and accepted 22 June 1995